MATH 2260 Calculus II
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Name: Answers
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Practice Quiz No. 2

Show all of your work, label your answers clearly, and do not use a calculator.

Problem 1 *Find the area of the surface generated by revolving the curve* $y = \sqrt{2x - x^2}$ *for* $\frac{1}{2} \leq x \leq \frac{3}{2}$ *around the x-axis.*

$$S = \int_{1/2}^{3/2} 2\pi \sqrt{2x - x^2} \sqrt{1 + \left(\frac{d}{dx}\left(\sqrt{2x-x^2}\right)\right)^2}\, dx = 2\pi \int_{1/2}^{3/2} \sqrt{2x-x^2} \sqrt{1 + \left(\left(\tfrac{1}{2}(2x-x^2)^{-1/2}(2\right.\right.}$$

$$= 2\pi \int_{1/2}^{3/2} \sqrt{2x-x^2} \sqrt{1 + \left(\frac{1-x}{\sqrt{2x-x^2}}\right)^2}\, dx = 2\pi \int_{1/2}^{3/2} \sqrt{2x-x^2} \sqrt{1 + \frac{1-2x+x^2}{2x-x^2}}\, dx$$

$$= 2\pi \int_{1/2}^{3/2} \sqrt{2x-x^2} \sqrt{\frac{2x-x^2+1-2x+x^2}{2x-x^2}}\, dx = 2\pi \int_{1/2}^{3/2} \sqrt{2x-x^2} \sqrt{\frac{1}{2x-x^2}}\, d$$

$$= 2\pi \int_{1/2}^{3/2} \sqrt{\frac{2x-x^2}{2x-x^2}}\, dx = 2\pi \int_{1/2}^{3/2} 1\, dx = 2\pi \left[x \right]_{1/2}^{3/2} = 2\pi$$

Problem 2 *Find the area of the surface generated by revolving the curve* $x = \frac{1}{3}y^{3/2} - y^{1/2}$ *for* $1 \leq y \leq 3$ *around the y-axis.*

Because the function $x = \frac{1}{3}y^{3/2} - y^{1/2}$ lies to the left of the y-axis we need to revolve $x = -\frac{1}{3}y^{3/2} + y^{1/2}$ around the y-axis instead.

$$S = \int_1^3 -2\pi \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right) \sqrt{1 + \left(\frac{d}{dy}\left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right)\right)^2}\, dy$$

$$= -2\pi \int_1^3 \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right) \sqrt{1 + \left(\tfrac{1}{2}y^{1/2} - \tfrac{1}{2}y^{-1/2}\right)^2}\, dy$$

$$= -2\pi \int_1^3 \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right) \sqrt{1 + \tfrac{1}{4}\left(y - 2 + \tfrac{1}{y}\right)}\, dy$$

$$= -2\pi \int_1^3 \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right) \sqrt{\tfrac{1}{4}\left(y + 2 + \tfrac{1}{y}\right)}\, dy$$

$$= -2\pi \int_1^3 \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right) \sqrt{\left(\tfrac{1}{2}\left(y^{1/2} + y^{-1/2}\right)\right)^2}\, dy$$

$$= -2\pi \int_1^3 \left(\tfrac{1}{3}y^{3/2} - y^{1/2}\right)\left(\tfrac{1}{2}\right)\left(y^{1/2} + y^{-1/2}\right) dy$$

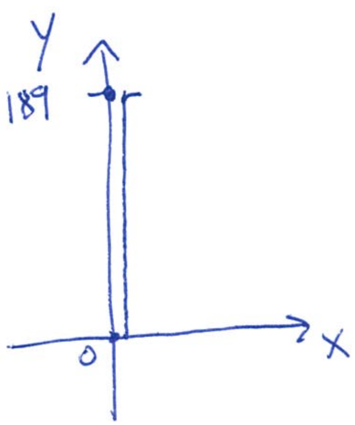
$$= -\pi \int_1^3 \tfrac{1}{3}y^2 + \tfrac{1}{3}y - 1 - y\, dy = -\pi \left[\tfrac{1}{3}\tfrac{y^3}{3} + \tfrac{1}{3}\tfrac{y^2}{2} - y - \tfrac{y^2}{2} \right]^3$$

Problem 3 *An electric elevator with a motor at the top has a cable weighing 4.5 lb/ft. When the car is at the first floor, 189 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. How much work does the motor do just lifting the cable when it takes the car from the first floor to the top?*

$$W = \int_0^{189} F(y)\,dy = \int_0^{189} (189 - y)(4.5)\,dy = 4.5\left[189y - \frac{y^2}{2}\right]_0^{189}$$

$$= 4.5\left(189^2 - \frac{189^2}{2}\right)$$

$$= 4.5\left(\frac{189^2}{2}\right)$$



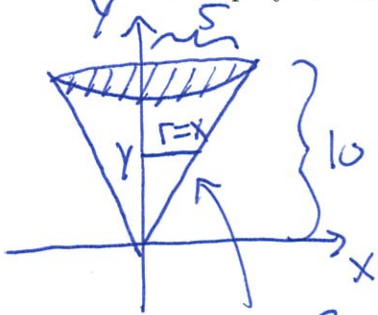
Problem 4 *A spring has a natural length of 9 in. A 500 lb force stretches the spring to 11 inches. How much work is done in stretching the spring from 9 in to 15 in?*

$$F = kx \Rightarrow 500 = k(11-9) \Rightarrow k = \frac{500}{2}$$

$$W = \int_0^6 F(x)\,dx = \int_0^6 \frac{500}{2}x\,dx = \frac{500}{2}\left[\frac{x^2}{2}\right]_0^6$$

$$= \frac{500}{4}(6^2)$$

Problem 5 *A conical tank with the pointed end facing down has a radius of 5 ft at the top and a height of 10 ft. How much work would it take to pump oil weighing 57 lb/ft^3 to the top of the tank if the tank is completely full?*



$y = 2x$

$x = \frac{1}{2}y$

$$W = \int_0^{10} \pi (x^2)\, 8\, (10-y)\, dy$$

$$= 57\pi \int_0^{10} \left(\frac{y}{2}\right)^2 (10-y)\, dy$$

$$= \frac{57\pi}{4} \int_0^{10} 10y^2 - y^3\, dy$$

$$= \frac{57\pi}{4} \left[10\frac{y^3}{3} - \frac{y^4}{4} \right]_0^{10}$$

Problem 6 *Solve the differential equation given by* $\frac{dy}{dx} = e^{6x-2y}$ *with* $y(0) = 4$.

$$\frac{dy}{dx} = e^{6x} e^{-2y}$$

$$\int e^{2y}\, dy = \int e^{6x}\, dx$$

$$\frac{1}{2}e^{2y} = \frac{1}{6}e^{6x} + C \quad , \quad y(0) = 4 \Rightarrow \frac{1}{2}e^{2(4)} = \frac{1}{6}e^{0} + C$$

$$\Rightarrow \frac{1}{2}e^{8} - \frac{1}{6} = C$$

$$\Rightarrow \frac{1}{2}e^{2y} = \frac{1}{6}e^{6x} + \left(\frac{1}{2}e^{8} - \frac{1}{6}\right)$$

$$\Rightarrow e^{2y} = 2\left[\frac{1}{6}e^{6x} + \frac{1}{2}e^{8} - \frac{1}{6}\right]$$

$$2y = \ln\left(2\left(\frac{1}{6}e^{6x} + \frac{1}{2}e^{8} - \frac{1}{6}\right)\right)$$

$$y = \frac{1}{2}\ln\left(2\left(\frac{1}{6}e^{6x} + \frac{1}{2}e^{8} - \frac{1}{6}\right)\right)$$

Problem 7 *Suppose the amount of oil pumped from an oil well decreases at the continuous rate of 10% per year. When will the well's output fall to one-fifth of its present value?*

Let the present value be y_0. Then after 1 year, $y(1) = .9y_0$. Using the exponential model, $y(t) = y_0 e^{kt}$, hence

$$.9y_0 = y(1) = y_0 e^{k(1)} = y_0 e^{k} \Rightarrow e^k = .9$$
$$\Rightarrow k = \ln(.9)$$

So $y(t) = y_0 e^{\ln(.9)t} = y_0 (.9)^t$

Set $\frac{1}{5} y_0 = y_0 (.9)^t \Rightarrow \frac{1}{5} = (.9)^t \Rightarrow \frac{1}{5} = e^{\ln(.9)t}$

$$\ln\left(\tfrac{1}{5}\right) = \ln(.9)\, t$$
$$\Rightarrow t = \frac{\ln(1/5)}{\ln(.9)}$$

Problem 8 *Evaluate the integral*

$$\int x^2 e^x dx$$

$$\int \underbrace{x^2}_{g(x)} \underbrace{e^x}_{f'(x)} dx = x^2 e^x - \int \underbrace{2x}_{v(x)} \underbrace{e^x}_{u'(x)} dx \qquad u(x) = e^x \quad v'(x) = 2$$

$f(x) = e^x$

$g'(x) = 2x$

$$= x^2 e^x - \left(2xe^x - \int 2e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2\int e^x dx$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x(x^2 - 2x + 2) + C$$

Problem 9 *Evaluate the integral*

$$\int x^2 \sin(x)dx$$

$$\int \underbrace{x^2}_{g(x)} \underbrace{\sin(x)}_{f'(x)} dx = -x^2\cos(x) + 2\int x\cos(x)\,dx = -x^2\cos(x) + 2\int \underbrace{x}_{v(x)} \underbrace{\cos(x)}_{u'(x)}\,dx$$

$$\boxed{f(x) = -\cos(x)\ ,\ g'(x) = 2x}$$

$u(x) = \sin(x)$

$v'(x) = 1$

$$\Rightarrow \int x^2\sin(x)\,dx = -x^2\cos(x) + 2\left(x\sin(x) - \int (1)\sin(x)\,dx\right)$$

$$= -x^2\cos(x) + 2\left(x\sin(x) + \cos(x)\right) + C$$

$$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$$

Problem 10 *Evaluate the integral*

$$\int (\log(x))^3 dx$$

$$\int \log^3(x)\,dx = \int \underbrace{\log(x)}_{f'(x)} \underbrace{\log^2(x)}_{g(x)}\,dx = \log^2(x)\left(x\log(x) - x\right) - \int (x\log(x) - x)(2\log(x$$

$$f(x) = x\log(x) - x\ ,\ g'(x) = 2\log(x)\frac{1}{x}$$

$$= x\log^3(x) - x\log^2(x) - 2\int \log^2(x) - \log(x)\,dx$$

$$= x\log^3(x) - x\log^2(x) - 2\left(\int \underbrace{\log(x)}_{u'(x)} \underbrace{\log(x)}_{v(x)}\,dx - \int \log(x)\,dx\right)$$

$$u(x) = x\log(x) - x\ ,\ v'(x) = \frac{1}{x}$$

$$= x\log^3(x) - x\log^2(x) - 2\left(\left(\log(x)(x\log(x) - x) - \int \log(x) - 1\,dx\right) - (x\log(x) -$$

$$= x\log^3(x) - \ldots$$

Problem 11 *Evaluate the integral*

$$\int \cos(\log(x))dx$$

$$\int \underbrace{(1)}_{f'(x)} \underbrace{\cos(\log(x))}_{g(x)} dx = x\cos(\log(x)) - \int \cancel{x}\left(-\sin(\log(x))\right)\frac{1}{\cancel{x}}dx$$

$$= x\cos(\log(x)) + \int \sin(\log(x))dx$$

$$f(x) = x \quad g'(x) = -\sin(\log(x))\frac{1}{x}$$

$$\Rightarrow \int \cos(\log(x))dx = x\cos(\log(x)) + \int \underbrace{(1)}_{u'(x)} \underbrace{\sin(\log(x))}_{v(x)} dx$$

$$u(x) = x \quad , \quad v'(x) = \cos(\log(x))\frac{1}{x}$$

$$= x\cos(\log(x)) + \left(x\sin(\log(x)) - \int x\cos(\log(x))\frac{1}{x}dx\right)$$

$$= x\cos(\log(x)) + x\sin(\log(x)) - \int \cos(\log(x))dx$$

$$\Rightarrow 2\int \cos(\log(x))dx = x\left(\cos(\log(x)) + \sin(\log(x\right.$$

$$\Rightarrow \int \cos(\log(x))dx = \frac{x}{2}\left(\cos(\log(x)) + \sin(\log(x))\right) + C$$